Correction CC1 de Mécanique 2017-2018

Exercice 1 *(3,5 points)*

a) $f_1'(t) = (2\omega t+2\omega\varphi)\sin((\omega t+\varphi)^2)$

$[\omega]=T^{-1}$ $[\varphi]=1$

b) $f_2'(t) = \frac{\frac{1}{\tau}\left(\frac{t}{\tau}-4\right)-\frac{1}{\tau}\left(\frac{t}{\tau}-1\right)}{\left(\frac{t}{\tau}+4\right)^2}$

$f_2'(t) = \frac{3}{\tau(\frac{t}{\tau}+4)^2}$

$[\tau]=T$

c) $f_3'(t) = \alpha(-\omega\sin(\omega t+\beta)\exp(-\gamma t^2)-2\gamma t\cos(\omega t+\beta)\exp(-\gamma t^2))$

$f_3'(t) = -\alpha\exp(-\gamma t^2)(\omega\sin(\omega t+\beta)+2\gamma\cos(\omega t+\beta)$

$[\alpha]=1$ $[\beta]=1$ $[\omega]=T^{-1}$ $[\gamma]=T^{-2}$

d) $f_4'(t) = \frac{\sqrt{3t^2+1}-\frac{t*6t}{2\sqrt{3t^2+1}}}{3t^2+1}$

$f_4'(t) = \frac{2}{(6t^2+2)\sqrt{3t^2+1}}$

e) $f_5'(t) = \frac{1}{\tau}\cos\left(\frac{\tau}{t}\right)-\frac{\tau}{t^2}*\frac{1}{t}\sin\left(\frac{\tau}{t}\right)$

$f_5'(t) = \frac{1}{\tau}\cos\left(\frac{\tau}{t}\right)-\frac{1}{t}\sin\left(\frac{\tau}{t}\right)$

$[\tau]=T$

f) $f_6'(t) = \frac{2\alpha\omega\tan(\omega t+\varphi)}{\cos^2(\omega t+\varphi)}$

$[\alpha]=1$ $[\omega]=T^{-1}$ $[\varphi]=1$

Exercice 2 *(2 points)*

Cherchons les dimensions des variables :

- $[T^2]=T^2$
- $[4\pi^2]=1$
- $[M+m]=M$

- On sait que $F_G = G*(m_1*m_2)/d^2$
  $[F_G]=[G]*[m_1*m_2]/[d^2]$
  $[G]=[ma]*[d^2]/[m_1*m_2]$
  $[G]=MLT^{-2}*L^2*M^{-2}$
  $\boxed{[G]=L^3M^{-1}T^{-2}}$

On a donc :

$$[T^2] = \frac{[4\pi^2]}{[G][M+m]}[a]^3$$

$$T^2 = \frac{1}{L^3M^{-1}T^{-2}M}[a]^3$$

$$[a]^3 = T^2L^3M^{-1}T^{-2}M$$

$$[a]^3 = L^3$$

$$\boxed{[a] = L}$$

<u>Exercice 3</u> *(3,5 points)*

1. $\boxed{\begin{array}{l}\vec{u} = -\cos(\alpha)\vec{\imath} + \sin(\alpha)\vec{\jmath} \\ \vec{v} = \sin(\alpha)\vec{\imath} + \cos(\alpha)\vec{\jmath}\end{array}}$

2. $\vec{u} + \vec{v} + \vec{w} = \vec{0}$
   $-\cos(\alpha)\vec{\imath} + \sin(\alpha)\vec{\jmath} + \sin(\alpha)\vec{\imath} + \cos(\alpha)\vec{\jmath} + x\vec{\imath} + y\vec{\jmath} = \vec{0}$
   $(x + \sin(\alpha)-\cos(\alpha))\vec{\imath} + (y + \cos(\alpha)+ \sin(\alpha))\vec{\jmath} = \vec{0}$
   $\begin{cases} x + \sin(\alpha) - \cos(\alpha) = 0 \\ y + \cos(\alpha) + \sin(\alpha) = 0 \end{cases}$
   $\begin{cases} x = \cos(\alpha) - \sin(\alpha) \\ y = -\cos(\alpha) - \sin(\alpha) \end{cases}$

   $\boxed{\vec{w} = (\cos(\alpha) - \sin(\alpha))\vec{\imath} - (\cos(\alpha) + \sin(\alpha))\vec{\jmath}}$

   - $\vec{u}.\vec{v} = -\cos(\alpha)\sin(\alpha) + \sin(\alpha)\cos(\alpha)$

     $\boxed{\vec{u}.\vec{v} = 0}$

   - $\vec{u}.\vec{w} = -\cos(\alpha)(\cos(\alpha) - \sin(\alpha)) - \sin(\alpha)(\cos(\alpha) + \sin(\alpha))$
     $\vec{u}.\vec{w} = -(\cos^2(\alpha) + \sin^2(\alpha))$

     $\boxed{\vec{u}.\vec{w} = -1}$

3. $||\vec{w}|| = \sqrt{x^2 + y^2}$
   $||\vec{w}|| = \sqrt{(\cos(\alpha) - \sin(\alpha))^2 + (\cos(\alpha) + \sin(\alpha))^2}$
   $||\vec{w}|| = \sqrt{2(\cos^2(\alpha) + \sin^2(\alpha))}$

   $\boxed{||\vec{w}|| = \sqrt{2} \neq 1}$
   Donc $\vec{w}$ n'est pas un vecteur unitaire

4. $\vec{u} \times \vec{v} = \begin{pmatrix} -cos(\alpha) \\ sin(\alpha) \\ 0 \end{pmatrix} \times \begin{pmatrix} sin\ (\alpha) \\ cos\ (\alpha) \\ 0 \end{pmatrix}$

$\vec{u} \times \vec{v} = \begin{pmatrix} 0 \\ 0 \\ -cos^2(\alpha)\text{-}sin^2(\alpha) \end{pmatrix}$

$\vec{u} \times \vec{v} = (-cos^2(\alpha)\text{-}sin^2(\alpha))\vec{k}$

$\boxed{\vec{u} \times \vec{v} = -\vec{k}}$

5. • $\vec{u} \times (\vec{v} \times \vec{w}) = \begin{pmatrix} -cos(\pi) \\ sin(\pi) \\ 0 \end{pmatrix} \times (\begin{pmatrix} sin\ (\pi) \\ cos\ (\pi) \\ 0 \end{pmatrix} \times \begin{pmatrix} cos(\pi)\text{ - }sin(\pi) \\ -cos(\pi)\text{- }sin(\pi) \\ 0 \end{pmatrix})$

$\vec{u} \times (\vec{v} \times \vec{w}) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times (\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix})$

$\vec{u} \times (\vec{v} \times \vec{w}) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\vec{u} \times (\vec{v} \times \vec{w}) = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$

$\boxed{\vec{u} \times (\vec{v} \times \vec{w}) = -\vec{j}}$

• $(\vec{u} \times \vec{v}) \times \vec{w} = (\begin{pmatrix} -cos(\pi) \\ sin(\pi) \\ 0 \end{pmatrix} \times \begin{pmatrix} sin\ (\pi) \\ cos\ (\pi) \\ 0 \end{pmatrix}) \times \begin{pmatrix} cos(\pi)\text{ - }sin(\pi) \\ -cos(\pi)\text{- }sin(\pi) \\ 0 \end{pmatrix}$

$(\vec{u} \times \vec{v}) \times \vec{w} = (\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}) \times \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$(\vec{u} \times \vec{v}) \times \vec{w} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$(\vec{u} \times \vec{v}) \times \vec{w} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\boxed{(\vec{u} \times \vec{v}) \times \vec{w} = \vec{i} + \vec{j}}$

Exercice 4 *(4 points)*

1. • $M_0 \begin{cases} x(0) = ae^{\omega\times 0}sin(\omega\times 0) \\ y(0) = ae^{\omega\times 0}cos(\omega\times 0) \\ z(0) = ae^{\omega\times 0} \end{cases}$

$\boxed{M_0 \begin{cases} x(0) = 0 \\ y(0) = a \\ z(0) = a \end{cases}}$

• $\lim_{t\to+\infty} z(t) = a \lim_{t\to+\infty} e^{\omega t} = +\infty$

La coordonnée *z(t)* du point M s'éloigne à l'infini au cours du temps donc M s'éloigne à l'infini au cours du temps

2. $\vec{v}(t) \begin{cases} v_x(t) = a(\omega e^{\omega t} sin(\omega t) + \omega e^{\omega t} cos(\omega t)) \\ v_y(t) = a(\omega e^{\omega t} cos(\omega t) - \omega e^{\omega t} sin(\omega t)) \\ v_z(t) = a\omega e^{\omega t} \end{cases}$

$$\boxed{\vec{v}(t) \begin{cases} v_x(t) = a\omega e^{\omega t}(sin(\omega t) + cos(\omega t)) \\ v_y(t) = a\omega e^{\omega t}(cos(\omega t) - sin(\omega t)) \\ v_z(t) = a\omega e^{\omega t} \end{cases}}$$

3. • $\vec{a}(t) \begin{cases} a_x(t) = a\omega(\omega e^{\omega t}(sin(\omega t) + cos(\omega t)) + e^{\omega t}(\omega cos(\omega t) - \omega sin(\omega t)) \\ a_y(t) = a\omega(\omega e^{\omega t}(cos(\omega t) - sin(\omega t)) + e^{\omega t}(-\omega sin(\omega t) - \omega cos(\omega t)) \\ a_z(t) = a\omega \times \omega e^{\omega t} \end{cases}$

$$\vec{a}(t) \begin{cases} a_x(t) = a\omega^2 e^{\omega t}(sin(\omega t) + cos(\omega t) + cos(\omega t) - sin(\omega t)) \\ a_y(t) = a\omega^2 e^{\omega t}(cos(\omega t) - sin(\omega t) - sin(\omega t) - cos(\omega t)) \\ a_z(t) = a\omega^2 e^{\omega t} \end{cases}$$

$$\boxed{\vec{a}(t) \begin{cases} a_x(t) = 2a\omega^2 e^{\omega t} cos(\omega t) \\ a_y(t) = -2a\omega^2 e^{\omega t} sin(\omega t) \\ a_z(t) = a\omega^2 e^{\omega t} \end{cases}}$$

• $||\vec{a}(t)|| = \sqrt{(2a\omega^2 e^{\omega t} cos(\omega t))^2 + (-2a\omega^2 e^{\omega t} sin(\omega t))^2 + (a\omega^2 e^{\omega t})^2}$

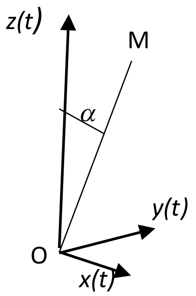
$||\vec{a}(t)|| = \sqrt{4a^2\omega^4 e^{2\omega t} cos^2(\omega t) + 4a^2\omega^4 e^{2\omega t} sin^2(\omega t) + a^2\omega^4 e^{2\omega t}}$

$||\vec{a}(t)|| = a\omega^2 e^{\omega t}\sqrt{4(cos^2(\omega t) + sin^2(\omega t)) + 1}$

$||\vec{a}(t)|| = a\omega^2 e^{\omega t}\sqrt{4 + 1}$

$$\boxed{||\vec{a}(t)|| = a\omega^2 e^{\omega t}\sqrt{5}}$$

4.



$cos(\alpha) = \frac{z(t)}{OM}$

$\alpha = Arcos(\frac{z(t)}{OM})$

$\alpha = Arcos(\frac{ae^{\omega t}}{\sqrt{(ae^{\omega t}\sin(\omega t))^2 + (ae^{\omega t}\cos(\omega t))^2 + (ae^{\omega t})^2}})$

$\alpha = Arcos(\frac{ae^{\omega t}}{\sqrt{a^2 e^{2\omega t}(sin^2(\omega t) + cos^2(\omega t) + 1)}})$

$\alpha = Arcos(\frac{1}{\sqrt{2}})$

$$\boxed{\alpha = \frac{\pi}{4} \text{ ou } \alpha = -\frac{\pi}{4}}$$

Exercice 5 *(4 points)*

1. • $\overrightarrow{OM} = r\overrightarrow{u_r}$
   $\overrightarrow{OM} = a(1 + cos(\theta))\overrightarrow{u_r}$

   • $\vec{v} = \dot{r}\overrightarrow{u_r} + r\omega\overrightarrow{u_\theta}$
   $\vec{v} = -a\omega sin(\theta)\overrightarrow{u_r} + a(1+cos(\theta))\omega\overrightarrow{u_\theta}$

$$\boxed{\vec{v} = a\omega(-sin(\theta)\overrightarrow{u_r} + (1+cos(\theta))\overrightarrow{u_\theta})}$$

2. $||\vec{v}|| = a\omega\sqrt{sin^2(\theta) + (1 + cos(\theta))^2}$
   $||\vec{v}|| = a\omega\sqrt{sin^2(\theta) + 1 + 2cos(\theta) + cos^2(\theta)}$

$$\boxed{||\vec{v}|| = a\omega\sqrt{2 + 2cos(\theta)}}$$

3. $\vec{a} = \ddot{r}\overrightarrow{u_r} + \omega\dot{r}\overrightarrow{u_\theta} + \omega\dot{r}\overrightarrow{u_\theta} - \omega^2 r\overrightarrow{u_r}$
   $\vec{a} = (\ddot{r} - \omega^2 r)\overrightarrow{u_r} + 2\omega\dot{r}\overrightarrow{u_\theta}$
   $\vec{a} = (\ddot{r} - \omega^2 r)\overrightarrow{u_r} + 2\omega\dot{r}\overrightarrow{u_\theta}$
   $\vec{a} = (-\omega^2 acos\,(\theta) - \omega^2(a(1 + cos(\theta))))\overrightarrow{u_r} - 2\omega^2 asin(\theta)\overrightarrow{u_\theta}$
   $\vec{a} = (-2\omega^2 acos\,(\theta) - \omega^2 a)\overrightarrow{u_r} - 2\omega^2 asin(\theta)\overrightarrow{u_\theta}$

$$\boxed{\vec{a} = -a\omega^2((2\,cos(\theta) + 1)\overrightarrow{u_r} + 2sin(\theta)\overrightarrow{u_\theta})}$$

Exercice 6 *(3 points)*

1. $[a]=[A]=[B]T^4$
   $[a]=LT^{-2}$

   - $\boxed{[A]=LT^{-2}}$

   - $[B]T^4=LT^{-2}$

     $\boxed{[B]=LT^{-6}}$

2. $\boxed{\vec{v}(t) = \frac{ds(t)}{dt}\vec{T}}$

3. $\boxed{\vec{a}(t) = \frac{d^2s(t)}{dt^2}\vec{T} + \frac{1}{R}(\frac{ds(t)}{dt})^2\vec{N}}$ avec *R* le rayon de courbure

4. On a: $\vec{a}(t) = A\vec{T} + Bt^4\vec{N}$

Par identification : $\frac{d^2s(t)}{dt^2}=A$

$\frac{ds(t)}{dt}=At+0$ car $v(0)=\frac{ds(0)}{dt}=0$

$s(t)=\frac{A}{2}t^2+0$ car $s(0)=0$

$$\boxed{s(t)=\frac{A}{2}t^2}$$

5. Par identification : $\frac{1}{R}(\frac{ds(t)}{dt})^2=Bt^4$

$R(s)=\frac{1}{Bt^4}(\frac{ds(t)}{dt})^2$

$R(s)=\frac{1}{B\frac{4s^2(t)}{A^2}}(\frac{ds(t)}{dt})^2$ car $t^2=\frac{2s(t)}{A}$

$R(s)=\frac{A^2}{B4s^2(t)}A^2\frac{2s(t)}{A}$ car $\frac{ds(t)}{dt}=A\sqrt{\frac{2s(t)}{A}}$

$R(s)=\frac{A^4 2s(t)}{AB4s^2(t)}$

$$\boxed{R(s)=\frac{A^3}{2Bs(t)}}$$